Analysis of Problem-Solving Ability Based on Polya Stages of Open University Students in Mathematics Courses

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Abstract: This type of research is qualitative research using a descriptive approach. The purpose of this research is to describe the problem-solving skills of the Polya stages in Mathematics courses. The subjects of this study were 15 students in the 3rd semester of the Open University, Elementary School Teacher Education Study Program. They were divided into three categories of problem-solving abilities, namely 6 students (40%) in the high category, 6 students (40%) in the medium category, and 3 students (20%) low category. Data related to problem-solving abilities according to Polya's stages were obtained from diagnostic tests and interviews. Subjects were taken by purposive sampling technique and data validation using triangulation. Data analysis techniques used data reduction, data presentation, and conclusion. The results of this study were 12 students (80%) who were in the high and medium categories were able to write down what was known and asked about the questions correctly, present the sequence of steps for solving using the proper steps or procedures and get the correct results, and write conclusions without writing checks. Whereas 3 students (20%) in the low category incompletely wrote what was known and asked about the questions, the sequence of solutions presented was inaccurate, wrote the procedures or steps but were incorrect, and wrote conclusions with the answers given were incorrect.

Keywords: Problem-Solving; Mathematic; Understanding; and Planning

A. Introduction

The rapid development in today's technology is also underpinned by developments in mathematics in various fields, such as number theory, algebra, analysis, probability theory, and discrete mathematics. To master and create technology in the future, strong mastery of mathematics is needed from an early age. Therefore, mathematics is one of the important subjects that must be taught in schools, starting from the basic education level to the higher education level (Chang, Y. L., & Huang, Y. I., 2014). According to Abdurrahman (2003) Mathematics is a symbolic language whose practical function is to express quantitative and spatial relations while its function is to facilitate thinking. In education, students’ abilities are honed through problems, so that students are able to improve the various competencies they have.

One of the objectives of learning mathematics is to develop problem solving abilities. This indicates that problem solving is one of the most important abilities to be honed in learning mathematics (Kristianti, N. K. H., Sudhita, I. W. R. S., & Riastini, P. N.). Öztürk et al (2020) states that problem solving is the process of solving problems that are resolved by using
information, skills, and also attitudes that are used when someone faces unfamiliar or unfamiliar situations. Sugiman (2009) states that problem-solving skills are not just the goal of learning mathematics but are even the heart of mathematics. This means, problem solving ability is a basic ability in learning mathematics.

Polya (1973) interprets problem solving as an attempt to find a way out of a difficulty in order to achieve a goal that is not so easy to achieve immediately. It appears that learning to solve problems is essentially learning to think or learning to reason, namely thinking or reasoning by applying previously acquired knowledge to solve new problems that have never been encountered (Purwanto, 2013). Through the process of solving problems, students can develop skills in critical thinking (Purwanto, 2013).

Polya (1973) states that there are four steps to the problem solving phase, namely analyzing and understanding the problem, designing and planning a solution, solving the problem, and re-checking all the steps taken. has been done (verifying a solution). Understanding the problem is of course not just reading, but also digesting the material presented and understanding what is going on. In other words understanding the problem/reading the problem is an activity of identifying what is asked to be solved and the facts given. The activity of planning, the problem solver finds the relationship between the given data (which is known) and the unknown (which is asked). If the relationship between the two is not immediately obtained, the problem solver can use auxiliary problems so that a solution plan is obtained (Baiduri, 2015). At this stage it is also related to what strategy will be used. Implementing the plan relates to checking each stage of the plan that was made before. Rechecking activities are related to the correctness/certainty of the solutions obtained (Baiduri, 2015).


Based on the explanation above, this time the researcher will present a study on the analysis of the problem-solving ability of polya stages in solving comparative material in mathematics courses. This study will describe students' problem-solving abilities at each stage of problem-solving abilities. It is hoped that this research can be used as a basis for providing lecturer assistance to students who experience problems in the process of solving mathematical problems.

B. Methods

This type of research is qualitative research using a descriptive approach. According to Arikunto (2010) Descriptive research is research that is intended to investigate circumstances, conditions, situations, events, activities, etc., and the results are presented in the form of a research report. The purpose of this study is to describe the ability to solve mathematical problems using polya stages in comparative material. The subjects in this study were students
of the Open University Elementary School Teacher Education Study Program Semester 2 of the Academic Year 2021/2022.

The subjects in this study were 15 students who would later be grouped based on the category of solving mathematical problems. The following is a category of students' mathematical problem solving abilities used by Hermawati et al (2021) that is:

<table>
<thead>
<tr>
<th>Categories</th>
<th>Achievement Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>$75 &lt; P \leq 100$</td>
</tr>
<tr>
<td>Medium</td>
<td>$60 &lt; P \leq 75$</td>
</tr>
<tr>
<td>Low</td>
<td>$0 &lt; P \leq 60$</td>
</tr>
</tbody>
</table>

The data obtained from all 15 students will be used, while for the interview stage two students will be taken in each category of mathematical problem solving abilities. This research was conducted in December 2022 - February 2023. To validate the data in this study, the triangulation method was used. Data collection techniques in this study used diagnostic tests and interviews.

Where the diagnostic test is in the form of a description test for the Mathematics course on comparative material. The expected data is in the form of students' work on a diagnostic test in the form of a description test of 2 questions along with the steps for solving them according to the Polya stage. The purpose of the diagnostic test is to determine students' problem-solving skills in solving math problems. The results of student answers to the diagnostic test questions were analyzed based on the assessment rubric. The rubric for assessing mathematical problem-solving abilities in this study was adapted from Mufarida's rubric (2008). This rubric can be seen in Table 2.

<table>
<thead>
<tr>
<th>Rated Aspect</th>
<th>Reaction to the problem (problem)</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Understanding</td>
<td>Do not write down/do not mention what is known and what is asked of the questions.</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Just write down/mention what is known.</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Write down/mention what is known and what is asked from the questions inaccurately.</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Write down/mention what is known and what is asked of the questions correctly.</td>
<td>4</td>
</tr>
<tr>
<td>Planning</td>
<td>Does not present a sequence of solution steps.</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Presents a sequence of solution steps, but the presented sequence of solutions is not accurate.</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Presents the correct sequence of solving steps, but leads to the wrong answer.</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Presenting the correct sequence of completion steps and leading to the correct answer.</td>
<td>4</td>
</tr>
<tr>
<td>Solving</td>
<td>There is no solution at all.</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>There is a solution, but the procedure is unclear.</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Using certain correct procedures but incorrect answers.</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Using certain correct procedures and correct results.</td>
<td>4</td>
</tr>
<tr>
<td>Checking</td>
<td>Do not write checks and do not write conclusions.</td>
<td>1</td>
</tr>
</tbody>
</table>
Furthermore, for the data obtained through the interview method using interview guidelines. Interview guidelines are unstructured because researchers do not use guidelines that have been compiled in a complete and systematic way to collect data, but the guidelines used are only an outline of the problems to be asked (Sugiyono, 2016). The purpose of the interview was to clarify the results of the student's answers in accordance with the polya problem solving stages.

Data analysis techniques were carried out by data reduction, data presentation, conclusions and verification. In this study also the results of data analysis using a descriptive approach. Data reduction is done when taking the subject as data. The researcher gave a diagnostic test to determine the category of problem solving ability. Furthermore, the same data will be taken one of which is then conducted interviews with reduced subjects. If there is data that can provide information, then the data is used. Presentation of data is done with narrative text. Conclusions were drawn after the researchers triangulated the method between diagnostic tests and interviews. From the results of the conclusions, the results of problem solving abilities based on the polya stage will be obtained which are divided into high, medium and low categories.

C. Results and Discussion

This research was conducted on 15 students of the Open University Elementary School Teacher Education Study Program to determine students' problem solving abilities based on the Polya stage which were in the high, medium, and low categories. The following are categories of mathematical problem solving abilities obtained by students in completing mathematical problem solving ability tests in Table 3.

<table>
<thead>
<tr>
<th>Categories</th>
<th>Number of Students</th>
<th>Percentage (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>6</td>
<td>40</td>
</tr>
<tr>
<td>Medium</td>
<td>6</td>
<td>40</td>
</tr>
<tr>
<td>Low</td>
<td>3</td>
<td>20</td>
</tr>
<tr>
<td>Total</td>
<td>15</td>
<td>100</td>
</tr>
</tbody>
</table>

Based on table 3 data, out of a total of 15 students who were the subject of the study, there were 6 students who had high category problem solving abilities with a percentage of 40%, 6 students had moderate category problem solving abilities with a percentage of 40%, and 3 students had problem solving abilities. low category problems with a percentage of 20%. Therefore, from all research subjects it was found that students with low category mathematical problem solving abilities had the least percentage compared to the high and medium categories.

Furthermore, the results of the recapitulation of the results of the ability assessment of mathematical problems in comparative material for PGSD UT students can be seen in Table 4.
The description of each stage of solving mathematical problem solving is described as follows:

1. **Understanding**

   The first step in solving a problem according to Polya is to understand the problem. In the first stage students are required to make a picture or illustration if possible, look for special cases, and try to understand the problem in a simple way. Furthermore, students are required to be able to write down/mention what is known and what is asked of the questions correctly. The following are the results of the Understanding stage analysis for each category of research subject.

   **a. Low Category**

   Based on the results of diagnostic tests and interviews, the results of student work in the low category at stage 1 (understanding) were obtained in Figure 1.

   S31: "What is known is that Nadia (N), Rafli (R), and Vito (V)'s test scores immediately add up the result 222"  

   **Figure 1. Results of Stage 1 Low Category Questions**

   Based on Figure 1, students are incomplete in writing down the information they know and are asked questions. This is also reinforced by the results of interviews where students can only determine the sum of the ages of the three students without defining one by one.

   **b. Medium Category**

   Based on the results of diagnostic tests and interviews, the results of student work in the medium category at stage 1 (understanding) were obtained in Figure 2.

   S21: "It is known that Rafli’s score is 2 higher than Nadia and Vito’s score is 7 lower than Rafli’s. The sum of the three ages of Rafli, Vito and Nadia is 222."

   P: “whereas what is being asked about?”  

   S21: “what is being asked is the grades of Rafli, Nadia and Vito.”  

   **Figure 2. Work Results of Stage 1 Questions Medium Category**
It can be seen from Figure 2 that students only wrote down what was known in the problem but did not write down exactly what was asked from the problem. However, during the interview students can explain what is asked in the questions.

c. **High Category**

Based on the results of diagnostic tests and interviews, the results of student work in the high category at stage 1 (understanding) were obtained in Figure 3.
Based on Figure 3 students can write down or mention what is known and asked in the questions correctly and completely. This is also reinforced during interviews where students can explain what is known and asked questions.

From the results of the analysis above, it was found that the three categories had different stage 1 (Understanding) abilities, where the low category could not write down or mention what was known and asked about the questions, but for the medium and high categories students were able to write down or mention what was known and asked questions.

Based on the test answers, it was found that 12 students (80%) could understand the problem in question number 1, while for question no.2 there were 10 students (66.67%). Most students cannot understand the problem because students have not been able to identify the elements that are known in the problem and change the statement into a mathematical model. This is in line with Timutius et al (2018) which states that students who do not identify the necessary elements as known or asked mean that students do not understand the problem properly.

2. Planning

The second step of problem solving according to Polya is to plan a settlement. The designing and planning phase includes planning solutions systematically, and determining what to do, how to do it and the expected results. At this stage, 2 types of responses were found by students, namely students writing a settlement plan on the answer sheet and students not writing a settlement plan on the answer sheet (doing directly). The following are the results of the analysis of the Planning stage for each category of research subjects.

a. Low Category

Based on the results of diagnostic tests and interviews, the results of student work in the low category in stage 2 (planning) were obtained as follows:

S31: "to first use the equation of the sum of the three values, then find the value of each."
b. Medium Category

Based on the results of diagnostic tests and interviews, the results of student work in the medium category in stage 2 (planning) were obtained in Figure 5.

![Figure 5. Work Results of Phase 2 Medium Category Questions](image)

S21: “To solve this problem, use the equation for the sum of the three values, which is 222, then enter the Vito and Nadia equations to get Rafli's value. In the end, Nadia and Vito's scores will be obtained.”

In Figure 5 it can be seen that students in the moderate category have been able to present the sequence of completion steps correctly, but lead to inaccurate answers because the completion steps are incomplete. Even though the results of the interviews showed that students were able to plan a settlement with the correct steps.

c. High Category

Based on the results of diagnostic tests and interviews, the results of student work in the high category in stage 2 (planning) were obtained as follows:

![Figure 6. Work Results of Stage 2 Questions in the High Category](image)

S11: "first use the equation of the sum of the three values \( R + N + V = 222 \). Then enter the equations \( N = R - 2 \) and \( V = R - 7 \) then the value of \( R \) or Rafli will be obtained. From the value of \( R = 77 \) included in the \( N \) and \( V \) equations, the Nadia and Vito values will be obtained.”

Based on Figure 6, students in the high category are able to present the sequence of completion steps correctly, completely and lead to the correct answer. This shows that students are able to make plans for the problems they face with reinforcement from the results of interviews where students can explain the settlement plan properly according to what is written.

From the results of the analysis above, it was found that the three categories had different stage 2 (Planning) capabilities, where the low category could not present the sequence of completion steps, the medium category could present the sequence of steps correctly but led to an incorrect answer, and the category height can present the correct, complete sequence of completion steps and lead to the correct answer.

Based on the test answers, it was found that 8 students (53.33%) were able to correctly present the order of completion steps in question number 1, while for question no.2 there were 10 students (66.67%). Most students are able to present the order of completion correctly, but have not yet led to the correct answer.
3. Solving

The third stage is to find a solution to the problem. In the problem solving stage it is very dependent on the experience of students to be more creative in compiling a solution to a problem. Following are the results of the analysis of the Solving stage for each category of research subjects.

a. Low Category

Based on the results of diagnostic tests and interviews, the results of student work in the low category in stage 3 (solving) were obtained as follows:

S31: "The sum of Nadia, Rafli and Vito's scores is 222. Then we get Rafli's score of 77 from 74 plus 2 and 1. Then Nadia's score is 75 from 74 plus 1 while Vito's score is 69 plus 1 to 70."

Figure 7. Results of Stage 3 Low Category Questions

Based on Figure 7 it appears that students in the low category provide solutions but the procedures or steps are not clear so that the answers given are also wrong. Students do not understand what steps to solve the problem.

b. Medium Category

Based on the results of the diagnostic tests and interviews, the results of student work in the medium category were obtained in stage 3 (solving) as follows:

S21: "first add up R + V + N = 222, then enter the equations V and N so you get the Rafli value of 77 then enter it into the equation to get the values of Nadia 75 and Vito 70."

Figure 8. Work Results of Phase 3 Medium Category Questions

Figure 8 shows that students are able to answer questions with the right flow and stages so as to provide the correct final result according to the plan in the previous step. This was also reinforced by the results of the interviews which explained the settlement procedures that were appropriate to the problem.

c. High Category

Based on the results of diagnostic tests and interviews, the results of student work in the low category in stage 3 (solving) were obtained in Figure 9.
S11: "first use the equation of the sum of the three values $R + N + V = 222$. Then enter the equations $N$ and $V$ where $N = R - 2$ and $V = R - 7$ then calculate the value $R = 77$. Enter the value $R = 77$ in the equation $N$ and $V$ so that each obtained 75 and 70."

Figure 9. Work Results of Stage 3 Questions in the High Category

In Figure 9 it appears that students in the high category are able to solve questions in a coherent and correct manner so that they get the correct final grade according to what was planned in the previous step.

Based on the analysis, it was found that the three categories had different stage 3 (Solving) abilities, where the low category provided solutions but the procedure or rarity was not clear so that the answers given were also wrong, the medium and high categories were able to answer questions with the right flow and stages so as to provide correct final result as planned in the previous step.

Based on the test answers, it was found that 12 students (80%) were able to answer the questions with the correct flow and steps so as to provide the correct final results according to plan in question number 1, while for question no.2 there were 10 students (66.67%). Most students are able to find strategies or solutions using the right procedures in solving problems so that they get the right answers. This is in line with Aspiandi et al (2020) which states that students with good problem-solving skills can find the right strategy or solution to solve problems.

4. Checking

The last stage is examining the solution which consists of activities using specific examinations of each information and completion steps and using general inspections to find out the problem in general and its development. In addition, at this stage it is equipped with writing conclusions on solving the problem. Following are the results of the Checking stage analysis for each category of research subjects.

a. Low Category

Based on the results of diagnostic tests and interviews, the results of student work in the low category at stage 4 (checking) were obtained as follows:

P: “After getting the answer, did you check your answer again?
S31: "check again ma'am but just don't check that it's just a rudimentary one"

Figure 10. Low Category Stage 4 Problem Work Results
Figure 10 shows that students have checked their answers again, but because they do not understand the steps for solving them, the answers given are not correct. However, students write conclusions from answers even though the answers given are not correct.

b. Medium Category

Based on the results of diagnostic tests and interviews, the results of student work in the medium category at stage 4 (checking) were obtained as follows:

P: "After getting the answers, did you check the answers again?"
S21: "Yes ma'am, I checked again from the beginning regarding the equations and calculations."
P: "to strengthen the answer what to do?"
S21: "I will write down the answers again in conclusion for each value of Rafli, Nadia and Vito."

Figure 11. Work Results of Phase 4 Medium Category Questions

Based on Figure 11 it appears that students in the middle category did not write down the re-checking stage but during the interview it was conveyed that the student checked his answers regarding equations and calculations. In addition, students also write conclusions about answers with the right results.

c. High Category

Based on the results of diagnostic tests and interviews, the results of student work in the high category at stage 4 (checking) were obtained as follows:

P: "After finishing work, the answers are checked again?"
S11: "Yes, I checked again from the beginning to the calculations, then I gave a conclusion for each value from Rafli, Nadia and Vito."

Figure 12. Work Results of Stage 4 Questions in the High Category

It can be seen from Figure 12 that students in the category did not write down the re-checking stage but during the interview it was conveyed that the student checked the steps and calculations. In addition, students also wrote a summary of the answers with the correct results.

Based on the analysis, it was found that the three categories had different stage 4 (Checking) abilities, where the low category gave but the conclusions that were written were not correct, the medium and high categories wrote the conclusions of the answers with the right results without writing down the checks.

Based on the test answers, it was found that 9 students (60%) were able to write a conclusion with the right results on question number 1, while for question no.2 there were 11 students (73.33%). Most of the students wrote the conclusion of the answers with the right results without writing checks.

Based on the results of the analysis above, students who fall into the high category have good problem-solving skills according to the Polya stage. This is in line with the results of
Christina & Adirakasiwi's research (2021) which states that students with high problem-solving abilities are able to use the four stages of Polya well. According to Christina & Adirakasiwi (2021) in their research it showed students with low mathematical problem solving abilities were more dominant than students with high and moderate mathematical problem solving abilities. The results of this study were different from this study, where as many as 12 students (80%) who were in the high and medium categories had good solving skills according to the Polya stage.

D. Conclusion

Based on the results of the analysis and discussion of this research, it can be concluded that problem solving abilities according to Polya vary at each stage in each category. For students in the low category, they are incomplete in writing down the information they know and are asked about the questions, the sequence of solutions presented is inaccurate, the procedures or steps are written but they are not correct, and the answers are checked again but the conclusions written are not correct.

Whereas students in the moderate category have the ability to solve problems, namely writing what is known in the problem but not writing down what is asked of the problem, presenting the correct sequence of steps for solving the problem, but leading to wrong answers using certain procedures or steps that are correct and the results are correct, and write down the conclusion of the answer with the right results without writing a check.

For the high category, according to Polya, the ability to solve problems is writing/mentioning what is known and what is asked of the questions correctly, presenting the correct sequence of steps for solving the problem, complete and leading to the right answer, using the right steps or procedures and getting results correct, and write conclusions without writing checks.

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